Ring-Segment Container

A ring-segment container has been shown in Figure 7(b). For this design, the equilibrium requirement, Equation (24), relates p_1 and p_2. Under shrink-fit it is assumed that the segments just barely contact each, i.e., the segments carry no hoop stress. (If the segments were in strong contact with each other then they would act like a complete ring, i.e., they would carry compressive hoop stress, and the distinction between a ring-segment container and a multi-ring container would be lost.) Thus, the same equilibrium requirement applies to the residual pressures q_1 and q_2. This requirement is

$$p \;= p_1/k_2, \qquad q \;= q_1/k_2 \tag{54a, b}$$

In order to determine the pressures p_1 and q_1 the following radial deformation equation is formulated:

$$\begin{aligned} u_2(r_2) - u_2(r_1) + \Delta_{12} + \alpha_2\Delta T\,(r_2 - r_1) &= u_3(r_2) - u_1(r_1) \\ &\quad + \alpha_3\Delta T r_2 - \alpha_1\Delta T r_1 \end{aligned} \tag{55}$$

where

Δ_{12} = the manufactured interference defined as the amount $(r_2 - r_1)$ of the segments exceeds $(r_2 - r_1)$ of the cylinders

$u_m(r_m)$ = the radial deformation of component n at r_m due to pressure p_n or q_n at r_n and p_{n-1} or q_{n-1} at r_{n-1}

α_n = thermal coefficient of expansion of component n

ΔT = temperature change from room temperature.

If the elasticity solutions, Equations (17a) and (22a), for the u_n, and Equation (54a) for p_2 are substituted into Equation (55) and the resulting expression solved for p_1, then there results

$$p_1 = \frac{1}{g}\left\{\frac{2p}{k_1{}^2-1} + 2\,\frac{E_1}{E}\,\frac{k_2k_3{}^2p_3}{(k_3{}^2-1)} + \frac{E_1\Delta_{12}}{r_1} - \Delta T E_1\left[\,k_2(\alpha_3 - \alpha_2) + (\alpha_2 - \alpha_1)\right]\right\} \tag{56}$$

where

$$\begin{aligned} g = \frac{k_1{}^2+1}{k_1{}^2-1} + \frac{E_1}{E_2}\left[\frac{2(k_2-1)}{k_2+1} + \frac{M_1}{\beta_1}\,(f_3(r_1) - k_2f_3(r_2))\right] \\ + \frac{E_1}{E_3}\left[\frac{k_3{}^2+1}{k_3 2-1} + \nu\right] - \nu \end{aligned} \tag{57}$$

The E_n are the moduli of elasticity at temperature. The parameters M_1 and β_1 and the function $f_3(r)$ have been defined previously in reference to Equations (22a, b). The procedure for finding q_1 is the same as that for finding p_1 except that $p = 0$ and q_3 replaces p_3, i.e.,

$$q_1 = \frac{1}{g}\left\{ 2\,\frac{E_1}{E}\,\frac{k_2 k_3{}^2 q_3}{(k_3{}^2-1)} + \frac{E_1 \Delta_{12}}{r_1} - \Delta T E_1 \left[k_2(\alpha_3-\alpha_2) + (\alpha_2-\alpha_1)\right]\right\} \tag{58}$$

A fatigue analysis of the high-strength liner is now conducted. The range in the hoop stress at the bore is:

$$(\sigma_\theta)_r = \frac{(\sigma_\theta)_{max} - (\sigma_\theta)_{min}}{2} = \frac{p}{2}\,\frac{(k_1{}^2+1)}{(k_1{}^2-1)} - \frac{(p_1-q_1)k_1{}^2}{k_1{}^2-1} \tag{59}$$

where Equation (16a) has been used. (p_1-q_1) is given by Equation (58), but an expression for (q_3-p_3) is needed before Equation (59) can be used to solve for p. The expression for (p_3-q_3) is obtained from Equation (35) with (p_2-q_2) replacing p and with $k_3{}^2 k_4{}^2 \ldots k_N{}^2$ replacing K^2 in Equation (34). There results

$$q_n = p_n - \frac{(p_2-q_2)\,(k_{n+1}{}^2\, k_{n+2}{}^2 \ldots k_N{}^2-1)}{(k_3{}^2\, k_4{}^2 \,\ldots\, k_N{}^2-1)}, \quad n \geqq 3 \tag{60}$$

Substituting for (q_3-p_3) from Equation (60) into (58), then substituting for (p_1-q_1) from Equation (58) into (59), equating $(\sigma_\theta)_r$ and $\alpha_r\sigma_1$ from Definition (13a), and solving for p/σ_1, one obtains

$$\frac{p}{\sigma_1} = \frac{2\alpha_r(k_1{}^2-1)^2(g-h)}{\left[(g-h)\,(k_1{}^4-1) - 4k_1{}^2\right]} \tag{61}$$

where

$$h = \frac{2E_1\, k_n{}^2\, (k_n{}^{2(N-3)}-1)}{E_3\, (k_n{}^{2(N-2)}-1)} \tag{62}$$

($k_3 = k_4 = \ldots = k_n$ for the outer cylinders as shown by Equation (48). Therefore, $k_3{}^2\, k_4{}^2 \ldots k_N{}^2 = k_n{}^{2(N-2)}$ in the expression for h.)

It is easily shown that (g-h) is independent of N, the number of components. Therefore, p/σ_1 given by Equation (61) is independent of N. However, p/σ_1 is dependent upon k_1 whereas for the multi-ring container it was not as previously shown by Equation (47). This dependence is also shown in Figure 16. From this figure it is evident that the ring-segment container cannot withstand as great a pressure as the multi-ring container if the over-all size is the same. This result is believed due to the fact that the segments do not offer any support to the liner – they are "floating" members between the liner and the third component, another ring. The effect is more pronounced as the segment size is increased. This is shown in Figure 17 where it is seen that the pressure decreases with increasing segment size.